On Intuitionistic Fuzzy Multisets Theory and Its Application in Diagnostic Medicine

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Abstract- Intuitionistic fuzzy multiset (IFMS) is an extension or a generalization of intuitionistic fuzzy set (IFS), and as such, very promising in decision mathematics. In this paper, some properties of intuitionistic fuzzy multiset (IFMS) are rigorously discussed. The application of IFMSs in diagnostic medicine via four distance measures are considered. Finally, a comparative analysis based on reliability and accuracy of the distance measures is considered in details.

Keywords-diagnostic medicine, fuzzy set, intuitionistic fuzzy set, fuzzy multiset, intuitionistic fuzzy multiset.

I. INTRODUCTION

Intuitionistic fuzzy set (IFS) introduced by Atanassov [1] as a generalization of fuzzy set proposed earlier in [2] received ample attentions in fuzzy community due to its flexibility, applicability and resourcefulness in tackling the issue of vagueness or the representation of imperfect knowledge in classical set theory. The main advantage of IFS is the capability to cope with the hesitancy that may exist. This is achieved by incorporating a second function, along with the membership function of the conventional fuzzy sets called non-membership function. In [3, 4, 5, 6, 7, 8, 9], rigorous research based on the theory and applications of intuitionistic fuzzy sets are carried out.

Notwithstanding, there are times that each element has different membership values with a corresponding non-membership values. Due to such situations, Shinoj and Sunil [10] introduced intuitionistic fuzzy multisets (IFMSs) from the combination of IFS and fuzzy multiset (FMS) proposed in [11] and showed its application in medical diagnosis. Obviously, IFMS is a generalized IFS or an extension of IFS [12]. Since inception of the notion of IFMS, many works have been conducted on the theory and applications of intuitionistic fuzzy sets are carried out.

Many applications of IFMS are carried out using distance measures approach. Distance measure between intuitionistic fuzzy multisets is an important concept in fuzzy mathematics because of its wide applications in real world, such as pattern recognition, machine learning, decision making and market prediction. Many distance measures between intuitionistic fuzzy multisets have been proposed and researched in [25, 26, 27, 28, 29, 30, 31, 32] as an extension of the works in [33, 34, 35, 36] in terms of IFS. Distance measure is a term that describes the difference between intuitionistic fuzzy multisets and can be considered as a dual concept of similarity measure.

This research studies the theory of intuitionistic fuzzy multisets and its application in diagnostic medicine. We make use of four distance measures of IFMS in [25] for the application and then, determine the reliability of the distance measures to ascertain the most accurate.
The rest of the paper is organized as follows: Section II briefly recalls some basic notions of fuzzy sets and IFSs. In Section III, the concept of IFMS is discussed and its properties are explained. The model of distance measures in terms of IFMSs are highlighted. Section IV discusses the application of IFMS to diagnostic medicine with a clear illustration via the highlighted distance measures, follows by the reliability analysis in Section V. Meanwhile, Section VI concludes the article.

II. PRELIMINARIES

Definition 1[2]. Let \( X \) be a nonempty set. A fuzzy set \( A \) drawn from \( X \) is defined as

\[ A = \{ (x, \mu_A(x)) : x \in X \} \]

where \( \mu_A(x) : X \rightarrow [0,1] \) is the membership function of the fuzzy set \( A \).

Definition 2[1]. Let \( X \) be a nonempty set. An intuitionistic fuzzy set \( A \) in \( X \) is an object having the form

\[ A = \{ (x, \mu_A(x), \nu_A(x)) : x \in X \} \]

where the functions \( \mu_A(x), \nu_A(x) : X \rightarrow [0,1] \) define respectively, the degree of membership and degree of non-membership of the element \( x \in X \) to the set \( A \), which is a subset of \( X \), and for every element \( x \in X \),

\[ 0 \leq \mu_A(x) + \nu_A(x) \leq 1. \]

Furthermore, we have \( \pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \) called the intuitionistic fuzzy set index or hesitation margin of \( x \) in \( A \). \( \pi_A(x) \) is the degree of non-determinacy of \( x \in X \) to the IFS \( A \) and \( \pi_A(x) \in [0,1] \) i.e., \( \pi_A(x) : X \rightarrow [0,1] \) for every \( x \in X \). The hesitation margin expresses the lack of knowledge of whether \( x \) belongs to IFS \( A \) or not.

Definition 3[11]. Let \( X \) be a nonempty set. A fuzzy multiset (FMS) \( A \) drawn from \( X \) is characterized by two functions: "count membership" of \( A \) denoted by \( \mathcal{CM}_A \) such that, \( \mathcal{CM}_A : X \rightarrow Q \), where \( Q \) is the set of all crisp multisets drawn from the unit interval \([0,1] \). Then for any \( x \in X \), the value \( \mathcal{CM}_A(x) \) is a crisp multiset drawn from \([0,1] \). For each \( x \in X \), the membership sequence is defined as the decreasingly ordered sequence of elements in \( \mathcal{CM}_A(x) \). It is denoted by \( (\mu_1^x(x), \mu_2^x(x), ..., \mu_n^x(x)) \), where \( \mu_i^x(x) \geq \mu_{i+1}^x(x) \geq ... \geq \mu_n^x(x) \).

III. INTUITIONISTIC FUZZY MULTISETS

Definition 4[10]. Let \( X \) be a nonempty set. An IFMS \( A \) drawn from \( X \) is characterized by two functions: "count membership" of \( A \) denoted as \( \mathcal{CM}_A \) and “count non-membership” of \( A \) denoted as \( \mathcal{CN}_A \) given respectively by \( \mathcal{CM}_A : X \rightarrow Q \) and \( \mathcal{CN}_A : X \rightarrow Q \) where \( Q \) is the set of all crisp multisets drawn from the unit interval \([0,1] \) such that for each \( x \in X \), the membership degrees of element in \( \mathcal{CM}_A(x) \) is defined as a decreasingly ordered sequence and it is denoted as \( (\mu_1^x(x), \mu_2^x(x), ..., \mu_n^x(x)) \) where \( \mu_1^x(x) \geq \mu_2^x(x) \geq ... \geq \mu_n^x(x) \) whereas the corresponding non-membership degrees of element in \( \mathcal{CN}_A(x) \) is denoted by \( (\nu_1^x(x), \nu_2^x(x), ..., \nu_n^x(x)) \) such that \( 0 \leq \nu_1^x(x) + \nu_2^x(x) + ... + \nu_n^x(x) \leq 1 \) for every \( x \in X \) and \( i = 1, ..., n \).

Let \( X \) be nonempty set. An IFMS \( A \) drawn from \( X \) is simply defined as

\[ A = \{ (\mu_1^x(x), \mu_2^x(x), ..., \mu_n^x(x)) : x \in X \}. \]

where the functions \( \mu_1^x(x), \mu_2^x(x), ..., \mu_n^x(x) : X \rightarrow [0,1] \) define the belongingness degrees and the non-belongingness degrees of \( A \) in \( X \) such that, \( 0 \leq \mu_i^x(x) + \nu_i^x(x) \leq 1 \) for \( i = 1, ..., n \). If the sequence of the membership functions and non-membership functions have only n-terms (i.e. finite), n is called the ‘dimension’ of \( A \).

Consequently \( A = \{ (\mu_1^x(x), \mu_2^x(x), ..., \mu_n^x(x)) : x \in X \} \) for \( i = 1, ..., n \) when no ambiguity arises, we simply write, \( A = \{ (\mu_i^x(x), \nu_i^x(x)) : x \in X \} \) for \( i = 1, ..., n \).
For each IFMS $A$ in $X$, $\mu_A^i(x) = 1 - \mu_A^i(x) - \nu_A^i(x)$ is the intuitionistic fuzzy multisets index or hesitation margin of $x$ in $A$. The hesitation margin $\nu_A^i(x)$ for each $i = 1, \ldots, \eta$ is the degree of non-determinacy for every $x \in X$, to the set $A$ and $\nu_A^i(x) \in [0,1]$. The function, $\nu_A^i(x)$ expresses lack of knowledge of whether $x$ in $X$ or not. In general, an IFMS $A$ is given as $A = \{(x, \mu_A^i(x), \nu_A^i(x), \pi_A^i(x)) | x \in X\}$, that is, $\mu_A^i(x) + \nu_A^i(x) + \pi_A^i(x) = 1$. We denote the set of all intuitionistic fuzzy multisets over $X$ as $IFMS(X)$.

**Definition 5[10].** The length of an element $x$ in an IFMS $A$ is defined as the cardinality of $\mathcal{C}_H(x)$ or $\mathcal{C}_N(x)$ for which $0 \leq \mu_A^i(x) + \nu_A^i(x) \leq 1$ and it is denoted by $L(x;A)$ i.e. the length of $x$ in $A$ for each $x \in X$. Then,

$$L(x;A) = |\mathcal{C}_H(x)| = |\mathcal{C}_N(x)|$$

**Definition 6[10].** If $A$ and $B$ are IFMSs drawn from $X$, then $L(x;A \ominus B) = \max \{L(x;A), L(x;B)\}$ or $L(x;A \oplus B) = \lor \{L(x;A), L(x;B)\}$ where $L(x;A) = L(x;A \ominus B)$ and $\lor$ denotes maximum.

**Remark 7.**

i. In an IFMS, $|\mathcal{C}_H(x)| = |\mathcal{C}_N(x)|$ for each $i = 1, 2, \ldots, \eta$.

ii. Whenever $\eta = 1$, an IFMS becomes IFS.

iii. IFMS and FMS of the same length have equal cardinality.

iv. Whenever the hesitation margin equals zero, an IFMS becomes FMS.

**Definition 8.** Let $X$ be a nonempty set and $A, B \in IFMS(X)$. Then we have

i. complement: $A^c = \{(x, \mathcal{C}_H(x), \mathcal{C}_N(x)) | x \in X\}$

ii. union: $A \cup B = \{(x, \max(\mu_A^i(x), \mu_B^i(x)), \min(\nu_A^i(x), \nu_B^i(x))) | x \in X\}$

iii. intersection: $A \cap B = \{(x, \min(\mu_A^i(x), \mu_B^i(x)), \max(\nu_A^i(x), \nu_B^i(x))) | x \in X\}$

iv. addition: $A \oplus B = \{(x, \mu_A^i(x) + \mu_B^i(x) - \mu_A^i(x)\mu_B^i(x), \nu_A^i(x)\nu_B^i(x)) | x \in X\}$

v. multiplication: $A \otimes B = \{(x, \mu_A^i(x)\mu_B^i(x), \nu_A^i(x) + \nu_B^i(x) - \nu_A^i(x)\nu_B^i(x)) | x \in X\}$

**Proposition 9.** Let $A, B, C \in IFMS(X)$ and $C \leq C$, then we have;

i. $A \otimes B \leq A \otimes C$

ii. $A \otimes B \leq A \otimes C$

iii. $A \otimes B \leq A \otimes C$

iv. $A \otimes B \leq A \otimes C$

Proof. Straightforward.

**Proposition 10.** Let $A, B \in IFMS(X)$, then if and only if $A = B$, we have

i. $A \otimes B = A$ or $A \otimes B = B$

ii. $A \otimes B = A$ or $A \otimes B = B$

Proof. Straightforward
Definition 11[12]. Let $X$ be nonempty. If $A$ is an IFMS drawn from $X$, then:

\[ A = \{ (x, \mu_A(x), \nu_A(x)) : x \in X \} = \{ (x, \mu_A(x), 1 - \mu_A(x)) : x \in X \} \]

\[ \forall A = \{ (x, 1 - \nu_A(x)) : x \in X \} = \{ (x, 1 - \nu_A(x), \nu_A(x)) : x \in X \} \]

for each $i = 1, 2, \ldots, n$.

Definition 12. Let $X$ be nonempty and $A \in IFMS(X)$. Then for any positive integer $n$, we have

\[ A^n = \{ (x, \mu_A^n(x), \nu_A^n(x)) : x \in X \} \]

and

\[ \forall A = \{ (x, 1 - [1 - \nu_A(x)]^n) : x \in X \} \]

for $i = 1, 2, \ldots, n$. If $\mu_A(x) = 0$, then $\mu_A^n(x) = 0$ and $\nu_A^n(x) = 1$.

Theorem 13. For any IFMSs $A$ and $B$ of a nonempty set $X$ such that $m, n$ are any two positive integers, then

i. $A^m = (\forall A)^m$

ii. $A^m = (\forall A)^m$

iii. if $m \geq n$, then $A^m \subseteq A^n$

iv. $\forall A = nA$

v. if $A \subseteq B$, then $A^m \subseteq B^m$

vi. if $A \subseteq B$, then $mA \subseteq nB$.

Proof.

i. Given that

\[ A = \{ (x, \mu_A(x), \nu_A(x)) : x \in X \} = \{ (x, \mu_A(x), 1 - \mu_A(x)) : x \in X \} \]

and

\[ A^n = \{ (x, [\mu_A(x)]^n, 1 - [\mu_A(x)]^n) : x \in X \} \]

then

\[ A^m = \{ (x, [\mu_A(x)]^m, 1 - [\mu_A(x)]^m) : x \in X \} \]

\[ \forall A = \{ (x, 1 - [1 - \nu_A(x)]^n) : x \in X \} \]

\[ \forall A = \{ (x, 1 - [1 - \nu_A(x)]^m) : x \in X \} \]

\[ = (\forall A)^m \]

ii. We know that

\[ \forall A = \{ (x, 1 - \nu_A(x)) : x \in X \} = \{ (x, 1 - \nu_A(x), \nu_A(x)) : x \in X \} \]

and

\[ A^n = \{ (x, [\mu_A(x)]^n, 1 - [\mu_A(x)]^n) : x \in X \} \]

then

\[ A^m = \{ (x, 1 - [1 - \nu_A(x)]^m) : x \in X \} \]

\[ = [\mu_A^n(x)]^m : x \in X \}

\[ = (\forall A)^m \]

iii. For $m \geq n$, $[\mu_A(x)]^m \leq [\mu_A(x)]^n$ and $1 - [1 - \nu_A(x)]^m \geq 1 - [1 - \nu_A(x)]^n$, then $A^m \subseteq A^n$.

iv. Where $\forall A = \{ (x, 1 - [1 - \nu_A(x)]^n) : x \in X \}$

\[ \forall A = \{ (x, 1 - [1 - \nu_A(x)]^n) : x \in X \} = n\forall A \]

v. Where $\forall A = \{ (x, 1 - [1 - \nu_A(x)]^m) : x \in X \}$

\[ \forall A = \{ (x, [\mu_A(x)]^n) : x \in X \} = m\forall A \]

vi. $A \subseteq B$ implies $\forall A \leq \forall B$ and $\forall B \geq \forall A$. Consequently, $[\mu_A(x)]^m \leq [\mu_B(x)]^n$ and $1 - [1 - \nu_A(x)]^m \geq 1 - [1 - \nu_B(x)]^n$. Hence, $A^m \subseteq B^m$. 

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vii. \( A \subseteq B \iff \forall x \in A \forall \alpha \in \mathcal{A} \left[ v^A_A(x) \leq v^A_B(x) \right] \) and \( \forall x \in B \forall \alpha \in \mathcal{A} \left[ v^B_B(x) \geq v^B_A(x) \right] \). Consequently, 
\[
1 - [1 - (1 - v^A_A(x))]^n \leq 1 - [1 - v^B_B(x)]^n \quad \text{and} \quad [v^A_A(x)]^n \geq [v^B_B(x)]^n
\]
then \( nA \subseteq nB \) for
\[
nA = \{ x : 1 - (1 - v^A_A(x))^n \geq 1 - (1 - v^B_B(x))^n \} \times X
\]

**Theorem 14.** For any IFMSs \( A \) and \( B \) of a nonempty set \( X \) such that \( n \) is any positive integer;

i. \( (A \cap B)^n = A^n \cap B^n \)

ii. \( (A \cup B)^n = A^n \cup B^n \)

iii. \( n(A \cap B) = nA \cap nB \)

iv. \( n(A \cup B) = nA \cup nB \)

**Proof:**

i. \( (A \cap B)^n = \{ x : 1 - (1 - v^A_A(x))^n \leq 1 - (1 - v^B_B(x))^n \} \times X \)

\[
= \{ x : \min \{[v^A_A(x)]^n, [v^B_B(x)]^n \} \leq \min \{[v^A_A(x)]^n, [v^B_B(x)]^n \} \} \times X
\]

\[
= \{ x : \min \{[v^A_A(x)]^n, [v^B_B(x)]^n \} \leq \min \{[v^A_A(x)]^n, [v^B_B(x)]^n \} \} \times X
\]

\[
= A^n \cap B^n.
\]

ii. \( (A \cup B)^n = \{ x : 1 - (1 - v^A_A(x))^n \leq 1 - (1 - v^B_B(x))^n \} \times X \)

\[
= \{ x : \max \{[v^A_A(x)]^n, [v^B_B(x)]^n \} \leq \max \{[v^A_A(x)]^n, [v^B_B(x)]^n \} \} \times X
\]

\[
= \{ x : \max \{[v^A_A(x)]^n, [v^B_B(x)]^n \} \leq \max \{[v^A_A(x)]^n, [v^B_B(x)]^n \} \} \times X
\]

\[
= A^n \cup B^n.
\]

iii. \( n(A \cap B) = \{ x : 1 - v^A_A(x)_A, v^B_B(x)_B \} \leq nA \cap nB \)

\[
= \{ x : \max \{[v^A_A(x)]^n, [v^B_B(x)]^n \} \leq \min \{[v^A_A(x)]^n, [v^B_B(x)]^n \} \} \times X
\]

\[
= \{ x : \max \{[v^A_A(x)]^n, [v^B_B(x)]^n \} \leq \min \{[v^A_A(x)]^n, [v^B_B(x)]^n \} \} \times X
\]

\[
= nA \cap nB.
\]

iv. \( n(A \cup B) = \{ x : 1 - v^A_A(x)_A, v^B_B(x)_B \} \leq nA \cup nB \)

\[
= \{ x : 1 - v^A_A(x)_A, v^B_B(x)_B \} \leq [v^A_A(x)]^n \times X
\]

\[
= \{ x : 1 - v^A_A(x)_A, v^B_B(x)_B \} \leq [v^B_B(x)]^n \times X
\]

\[
= nA \cup nB.
\]

**Theorem 15.** Let \( X \) be nonempty such that \( A, B, C \in \text{IFMS}(X) \). Then distance measure \( d \) is a mapping
\[
d : X \times X \rightarrow [0, 1]
\]
if \( d(A, B) \) satisfies the following axioms:

i. \( 0 \leq d(A, B) \leq 1 \)

ii. \( d(A, B) = 0 \) if and only if \( A = B \)

iii. \( d(A, B) = d(B, A) \) (i.e. symmetric)

iv. \( d(A, C) + d(B, C) \geq d(A, B) \)

v. if \( A \subseteq B \subseteq C \), then \( d(A, C) \geq d(A, B) \) and \( d(A, C) \geq d(B, C) \).
Then $d(A,B)$ is a distance measure between $A$ and $B$.

Let $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ and $B = \{(x, \mu_B(x), \nu_B(x)) : x \in X\}$ and $X = \{x_1, x_2, \ldots, x_n\}$ for $j = 1, 2, \ldots, n$. Then the following distances, measure the distance between $A$ and $B$ as follow.

The Hamming distance;

$$d(A,B) = \frac{1}{n} \sum_{x=1}^{n} \left[ |\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| + |\pi_A(x) - \pi_B(x)| \right]$$

The Euclidean distance;

$$d(A,B) = \sqrt{\frac{1}{2n} \sum_{x=1}^{n} \left[ (\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2 + (\pi_A(x) - \pi_B(x))^2 \right]}$$

The normalized Hamming distance;

$$d(A,B) = \frac{1}{2n} \sum_{x=1}^{n} \left[ |\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| + |\pi_A(x) - \pi_B(x)| \right]$$

The normalized Euclidean distance

$$d(A,B) = \sqrt{\frac{1}{2n} \sum_{x=1}^{n} \left[ (\mu_A(x) - \mu_B(x))^2 + (\nu_A(x) - \nu_B(x))^2 + (\pi_A(x) - \pi_B(x))^2 \right]}$$

**IV. APPLICATION OF INTUITIONISTIC FUZZY MULTISETS IN DIAGNOSTIC MEDICINE**

Most human reasoning involves the use of variables whose values are uncertain i.e. fuzzy sets. This is the basis for the concept of linguistic variable, that is, a variable with words values rather than numbers. But some cases like medical diagnosis, the description by a linguistic variable in terms of membership function alone is not sufficient because there is a chance of the existing of non-membership function. Then intuitionistic fuzzy set (IFS) can be used in such cases because it uses both membership and non-membership functions of an element in a set. Notwithstanding, there are situations that each element has different membership values and non-membership values as well. In such situations, intuitionistic fuzzy multiset (IFMS) can be used as a tool [10].

Let $P=\{P_1, P_2, P_3, P_4\}$ be a set of patients, $D=\{\text{viral fever, tuberculosis, typhoid, throat disease}\}$ be a set of diseases and $S=\{\text{temperature, cough, throat pain, headache, body pain}\}$ be a set of symptoms. From the basic knowledge of medicine, we assumed that Table 1 contains some diseases and their symptoms in intuitionistic fuzzy set values. We observed that, the diseases share the same symptoms but in different proportions.
TABLE 1

<table>
<thead>
<tr>
<th>DISEASES</th>
<th>VS SYMPTOMS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>temperature</td>
</tr>
<tr>
<td>viral fever</td>
<td>0.8,0.1,0.1</td>
</tr>
<tr>
<td>tuberculosis</td>
<td>0.2,0.7,0.1</td>
</tr>
<tr>
<td>typhoid</td>
<td>0.5,0.3,0.2</td>
</tr>
<tr>
<td>throat disease</td>
<td>0.1,0.7,0.2</td>
</tr>
</tbody>
</table>

In Table 1, each symptom is described by three numbers i.e. membership, non-membership and hesitation margin. For better diagnosis, we assumed that samples are taken from the patients in two different times (i.e. 7AM and 2PM) in a day. After the samples obtained at 7AM and 2PM have been examined, we get a supposed medical analysis of the patients as shown in Table 2.

TABLE 2

<table>
<thead>
<tr>
<th>PATIENTS VS SYMPTOMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>temperature</td>
</tr>
<tr>
<td>P1 1st</td>
</tr>
<tr>
<td>2nd</td>
</tr>
<tr>
<td>P2 1st</td>
</tr>
<tr>
<td>2nd</td>
</tr>
<tr>
<td>P3 1st</td>
</tr>
<tr>
<td>2nd</td>
</tr>
<tr>
<td>P4 1st</td>
</tr>
<tr>
<td>2nd</td>
</tr>
</tbody>
</table>

In Table 2, 1st and 2nd signify the results of the first test and second test, respectively. For easy calculation, we take the mean value i.e., \( \frac{1}{2} \) of each level as shown in Table 3.

TABLE 3

<table>
<thead>
<tr>
<th>PATIENTS VS SYMPTOMS USING MEAN VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>temperature</td>
</tr>
<tr>
<td>P1</td>
</tr>
<tr>
<td>P2</td>
</tr>
<tr>
<td>P3</td>
</tr>
<tr>
<td>P4</td>
</tr>
</tbody>
</table>

Using the distance measures above to calculate the distance between each of the patients in Table 3 and each of the diseases in Table 1 with respect to each of the symptoms, we get the following results as shown below.
From Table 4 to 7, patient P1 is diagnosed with typhoid, patient P2 is diagnosed with tuberculosis, patient P3 is diagnosed with throat disease and patient P4 is diagnosed with typhoid. If the distance between a patient and a particular disease is the shortest, the patient is likely to have the disease. All the distance measures give a harmonized diagnosis and none of the patients suffer from viral fever.

V. RELIABILITY ANALYSIS OF THE DISTANCE MEASURES

We use the table below to assess the reliability of the distance measures used in the medical diagnosis.

### TABLE 4
**DISTANCE BETWEEN PATIENTS AND DISEASES USING HAMMING DISTANCE**

<table>
<thead>
<tr>
<th>d(P,D)</th>
<th>viral fever</th>
<th>tuberculosis</th>
<th>typhoid</th>
<th>throat disease</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.19</td>
<td>0.43</td>
<td>0.15</td>
<td>0.42</td>
</tr>
<tr>
<td>P2</td>
<td>0.33</td>
<td>0.18</td>
<td>0.30</td>
<td>0.38</td>
</tr>
<tr>
<td>P3</td>
<td>0.40</td>
<td>0.32</td>
<td>0.30</td>
<td>0.18</td>
</tr>
<tr>
<td>P4</td>
<td>0.20</td>
<td>0.35</td>
<td>0.16</td>
<td>0.40</td>
</tr>
</tbody>
</table>

### TABLE 5
**DISTANCE BETWEEN PATIENTS AND DISEASES USING EUCLIDEAN DISTANCE**

<table>
<thead>
<tr>
<th>d(P,D)</th>
<th>viral fever</th>
<th>tuberculosis</th>
<th>typhoid</th>
<th>throat disease</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.1670</td>
<td>0.3977</td>
<td>0.1389</td>
<td>0.4037</td>
</tr>
<tr>
<td>P2</td>
<td>0.3091</td>
<td>0.1619</td>
<td>0.2771</td>
<td>0.3669</td>
</tr>
<tr>
<td>P3</td>
<td>0.3792</td>
<td>0.2965</td>
<td>0.2808</td>
<td>0.1729</td>
</tr>
<tr>
<td>P4</td>
<td>0.1822</td>
<td>0.3213</td>
<td>0.1409</td>
<td>0.3785</td>
</tr>
</tbody>
</table>

### TABLE 6
**DISTANCE BETWEEN PATIENTS AND DISEASES USING NORMALIZED HAMMING DISTANCE**

<table>
<thead>
<tr>
<th>d(P,D)</th>
<th>viral fever</th>
<th>tuberculosis</th>
<th>typhoid</th>
<th>throat disease</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.038</td>
<td>0.086</td>
<td>0.030</td>
<td>0.084</td>
</tr>
<tr>
<td>P2</td>
<td>0.066</td>
<td>0.036</td>
<td>0.060</td>
<td>0.076</td>
</tr>
<tr>
<td>P3</td>
<td>0.080</td>
<td>0.064</td>
<td>0.060</td>
<td>0.036</td>
</tr>
<tr>
<td>P4</td>
<td>0.040</td>
<td>0.070</td>
<td>0.032</td>
<td>0.080</td>
</tr>
</tbody>
</table>

### TABLE 7
**DISTANCE BETWEEN PATIENTS AND DISEASES USING NORMALIZED EUCLIDEAN DISTANCE**

<table>
<thead>
<tr>
<th>d(P,D)</th>
<th>viral fever</th>
<th>tuberculosis</th>
<th>typhoid</th>
<th>throat disease</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.0747</td>
<td>0.1779</td>
<td>0.0621</td>
<td>0.1806</td>
</tr>
<tr>
<td>P2</td>
<td>0.1382</td>
<td>0.0724</td>
<td>0.1239</td>
<td>0.1641</td>
</tr>
<tr>
<td>P3</td>
<td>0.1696</td>
<td>0.1326</td>
<td>0.1344</td>
<td>0.7732</td>
</tr>
<tr>
<td>P4</td>
<td>0.0815</td>
<td>0.1457</td>
<td>0.0630</td>
<td>0.1692</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>d(P,D)</th>
<th>viral fever</th>
<th>tuberculosis</th>
<th>typhoid</th>
<th>throat disease</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.1900</td>
<td>0.4300</td>
<td>0.1500</td>
<td>0.4200</td>
</tr>
<tr>
<td></td>
<td>0.1670</td>
<td>0.3977</td>
<td>0.1389</td>
<td>0.4037</td>
</tr>
<tr>
<td></td>
<td>0.0380</td>
<td>0.0860</td>
<td>0.0300</td>
<td>0.0840</td>
</tr>
<tr>
<td></td>
<td>0.0747</td>
<td>0.1779</td>
<td>0.0621</td>
<td>0.1806</td>
</tr>
<tr>
<td>P2</td>
<td>0.3300</td>
<td>0.1800</td>
<td>0.3000</td>
<td>0.3800</td>
</tr>
<tr>
<td></td>
<td>0.3091</td>
<td>0.1619</td>
<td>0.2771</td>
<td>0.3669</td>
</tr>
<tr>
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<td>0.0660</td>
<td>0.0360</td>
<td>0.0600</td>
<td>0.0760</td>
</tr>
<tr>
<td></td>
<td>0.1382</td>
<td>0.0724</td>
<td>0.1239</td>
<td>0.1641</td>
</tr>
<tr>
<td>P3</td>
<td>0.4000</td>
<td>0.3200</td>
<td>0.3000</td>
<td>0.1800</td>
</tr>
<tr>
<td></td>
<td>0.3792</td>
<td>0.2965</td>
<td>0.2808</td>
<td>0.1729</td>
</tr>
<tr>
<td></td>
<td>0.0800</td>
<td>0.0640</td>
<td>0.0600</td>
<td>0.0360</td>
</tr>
<tr>
<td></td>
<td>0.1696</td>
<td>0.1326</td>
<td>0.1344</td>
<td>0.0773</td>
</tr>
<tr>
<td>P4</td>
<td>0.2000</td>
<td>0.3500</td>
<td>0.1600</td>
<td>0.4000</td>
</tr>
<tr>
<td></td>
<td>0.1822</td>
<td>0.3213</td>
<td>0.1409</td>
<td>0.3785</td>
</tr>
<tr>
<td></td>
<td>0.0400</td>
<td>0.0700</td>
<td>0.0320</td>
<td>0.0800</td>
</tr>
<tr>
<td></td>
<td>0.0815</td>
<td>0.1437</td>
<td>0.0630</td>
<td>0.1692</td>
</tr>
</tbody>
</table>

From Table 8, the normalized Hamming distance gives a true and impeccable picture of the patients’ medical conditions since it provides the shortest distance from the patients to the diseases. The sequence of reliability and accuracy of the distance measures in decreasing order is thus: \( d_{\text{H}} < d_{\text{E}} < d_{\text{H,E}} < d_{\text{E}} \), such that \( d_{\text{H}} < d_{\text{E}} < d_{\text{H,E}} < d_{\text{H}} \).

VI. CONCLUSION

We have studied intuitionistic fuzzy multisets and deduced some relevant properties. The application of intuitionistic fuzzy multisets in diagnostic medicine is shown using Euclidean distance, Hamming distance, normalized Hamming distance and normalized Euclidean distance, respectively to calculate the distance between the patients and the diseases. Then, a comparison was made to determine the most accurate and reliable of the distance measures.

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REFERENCES


